

# Hybrid Quarkonium Masses up to the order of $\mathcal{O}(1/m_Q)$

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## Abstract

We have calculated the binding energy of the hybrid quarkonium up to the order of  $\mathcal{O}(1/m_Q)$  and found their decay constants scale like  $m_Q^{\frac{5}{4}}$  as  $m_Q \rightarrow \infty$ . The  $0^{--}$  and  $0^{++}$  hybrid quarkonium is exactly degenerate in the limit  $m_Q \rightarrow \infty$  while the  $\mathcal{O}(1/m_Q)$  correction renders the  $0^{--}$  mass lower than that for  $0^{++}$ . The  $1^{-+}$  and  $1^{+-}$  hybrid is nearly degenerate and lies 0.7 GeV lower than the  $0^{--}$ . If we use  $m_b = 4.8\text{GeV}$  the mass of the  $1^{-+} b\bar{b}g$  state is  $(10.75 \pm 0.20)$  GeV, which is the lightest exotic hybrid quarkonium.

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Quantum Chromodynamics (QCD) is believed to be the correct theory of strong interaction. In the massless light quark limit, QCD has exact  $SU_L(3) \times SU_R(3)$  symmetry. On the other hand, QCD has spin-flavor symmetry in the heavy quark limit  $m_Q \rightarrow \infty$ . In both cases we have an effective theory: the chiral perturbation theory ( $\chi$ PT) and the heavy quark effective theory (HQET).  $\chi$ PT is very useful for the low energy dynamics of strong interaction, while HQET provides a consistent framework to study hadrons containing one heavy quark. Symmetry plays a fundamental role in particle physics. It is important to explore novel features of the exotic system ( $Q\bar{Q}g$ ) with two heavy quarks in the limit  $m_Q \rightarrow \infty$ .

From quark model we know that a  $q\bar{q}$  meson with orbital angular momentum  $l$  and total spin  $s$  must have  $P = (-1)^{l+1}$  and  $C = (-1)^{l+s}$ . Thus a resonance with  $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \dots$  must be exotic. Such a state could be a multiquark state or gluonic excitation such as hybrids, glueballs. Very recently there appears experimental evidence for a light  $J^{PC} = 1^{-+}$  exotic [1, 2, 3, 4, 5]. The emergence of evidence for hybrids indicates the presence of dynamical glue in QCD.

The traditional theoretical approach to the gluonic excitations in mesons falls into two classes: the string or flux tube model and the constituent glue model with the glue confined by a bag or potential. In contrast, QCD sum rule (QSR) starts from the first principle like the lattice gauge theory and incorporates the nonperturbative effects via

various condensates in QCD vacuum. It has proven successful in extracting low-lying hadron masses and decay constants [6]. Several collaborations [7]-[9] have studied the masses of light hybrids with QCD sum rules. The mass of heavy hybrid quarkonium has been studied by one group [10]. But unfortunately the QCD sum rules for the exotic  $(Q\bar{Q}g)$  states with  $J^{PC} = 0^{--}, 1^{-+}$  are not stable, which prevents a reliable extraction of their masses [10]. Recently the masses and decay widths of hybrid mesons containing one heavy quark have been studied in the framework of HQET [11].

In this work we shall investigate the binding energy of hybrid quarkonium  $(Q\bar{Q}g)$ , the scaling behavior of their decay constants in the limit of  $m_Q \rightarrow \infty$  and their  $1/m_Q$  corrections. As we shall show later, all of our sum rules are stable after the heavy quark mass is separated out.

The interpolating current for the  $J^{PC} = 1^{-+}, 0^{++}$  heavy hybrid quarkonium reads

$$J_\mu(x) = \bar{Q}(x)g_s\gamma^\nu G_{\mu\nu}^a(x)\frac{\lambda^a}{2}Q(x), \quad (1)$$

and for the  $J^{PC} = 1^{+-}, 0^{--}$  ones

$$J_\mu^5(x) = \bar{q}(x)g_s\gamma^\nu\gamma_5 G_{\mu\nu}^a(x)\frac{\lambda^a}{2}h_v(x). \quad (2)$$

Denote the  $J^{PC} = 1^{-+}, 0^{++}, 1^{+-}, 0^{--}$  hybrid mesons by  $H_i$  and the decay constants by  $f_i$ ,  $i = 1, 2, 3, 4$ . For example,  $f_1$  is defined as:

$$\langle 0|J_\mu(0)|H_1\rangle = f_1\epsilon_\mu^1, \quad (3)$$

where  $\epsilon_\mu^1$  is the  $H_1$  polarization vector.

We consider the correlators

$$i \int d^4x e^{ipx} \langle 0|T\{J_\mu(x), J_\nu^\dagger(0)\}|0\rangle = -(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})\Pi_1(p^2) + \frac{p_\mu p_\nu}{p^2}\Pi_2(p^2), \quad (4)$$

$$i \int d^4x e^{ikx} \langle 0|T\{J_\mu^5(x), J_\nu^{5\dagger}(0)\}|0\rangle = -(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})\Pi_3(p^2) + \frac{p_\mu p_\nu}{p^2}\Pi_4(p^2). \quad (5)$$

The imaginary parts of  $\Pi_1, \Pi_2, \Pi_3, \Pi_4$  receive contributions from the  $H_1, H_2, H_3, H_4$  hybrid intermediate states respectively.

The dispersion relation for  $\Pi_i(p^2)$  reads

$$\Pi(p^2) = \int \frac{\rho(s)}{s - p^2 - i\epsilon} ds, \quad (6)$$

where  $\rho(s)$  is the spectral density.

At the phenomenological side

$$\Pi(p^2) = \frac{f^2}{M^2 - p^2} + \text{excited states} + \text{continuum}. \quad (7)$$

In order to derive the spectral density in full QCD we can either use the Cutkosky cutting rule or employ the dispersion relation and previous results of QCD sum rule for heavy quarkonium [12]. With both methods we have obtained the same results independently. For example, the spectral density in full QCD for  $H_1$  reads

$$\rho_1(s) = \frac{\alpha_s}{72\pi^3} \int_{4m_Q^2}^s \sqrt{1 - \frac{4m_Q^2}{x}} \left(1 + \frac{2m_Q^2}{x}\right) \frac{(s-x)^3(s+x)}{s^2} dx + \frac{\langle 0|g_s^2 G^2|0\rangle}{144\pi^2} \sqrt{1 - \frac{4m_Q^2}{s}} \left(1 + \frac{2m_Q^2}{s}\right), \quad (8)$$

which agrees with the expressions in [10].

Note the gluon condensate and the perturbative term is of the same order in the limit of  $m_Q \rightarrow \infty$ , which enables us to explore the scaling behavior. Let  $M = 2m_Q + \Lambda$ , where  $\Lambda$  is the leading order binding energy. Expanding (6), (7), (8) to the leading order of  $1/m_Q$  and making Borel transformation to suppress the continuum contribution, we arrive at

$$\frac{1}{4m_Q} f^2 e^{-\frac{\Lambda}{T}} = \int_0^{E_c} m_Q^{\frac{3}{2}} \rho_0(\epsilon) e^{-\frac{\epsilon}{T}} d\epsilon, \quad (9)$$

where  $\rho_0(\epsilon) = a_i \frac{\alpha_s}{\pi^3} \epsilon^4 \sqrt{\epsilon} + \frac{b_i}{\pi^2} < 0 | g_s^2 G^2 | 0 > \sqrt{\epsilon}$ ,  $\alpha_s = \frac{4\pi}{(11 - \frac{2}{3}n_f) \ln(\frac{E_c/2}{\Lambda_{\text{QCD}}})^2}$  and  $< 0 | g_s^2 G^2 | 0 > = 0.48 \text{ GeV}^4$ . The coefficients  $a_i$  etc are listed in TABLE I.  $E_c$  is the continuum threshold. Starting from  $E_c$  we have modeled the phenomenological spectral density with the parton-like one  $m_Q^{\frac{3}{2}} \rho_0(\epsilon)$ . The triple gluon condensate is suppressed by a factor of  $1/m_Q^2$  so it does not contribute up to the order  $\mathcal{O}(1/m_Q)$ . From (9) we can introduce a scaled decay constant  $F = m_Q^{-\frac{5}{4}} f$ , which remain constant in the limit  $m_Q \rightarrow \infty$ .

We further expand the hybrid mass  $M$  and decay constant  $f$  to the order of  $1/m_Q$ ,  $M = 2m_Q + \Lambda + \frac{\Lambda_1}{m_Q}$ ,  $f = m_Q^{\frac{5}{4}} (F + \frac{F_1}{m_Q})$ ,  $\Lambda_1$ ,  $F_1$  is the  $\mathcal{O}(1/m_Q)$  correction to  $\Lambda$  and  $F$  respectively. Similarly we have,

$$\frac{1}{4} [2F_1 F - \frac{F^2(\Lambda_1 + \frac{\Lambda^2}{4})}{T}] e^{-\frac{\Lambda}{T}} = \int_0^{E_c} \rho_1(\epsilon) e^{-\frac{\epsilon}{T}} d\epsilon, \quad (10)$$

where  $\rho_1(\epsilon) = c_i \frac{\alpha_s}{\pi^3} \epsilon^5 \sqrt{\epsilon} + \frac{d_i}{\pi^2} < 0 | g_s^2 G^2 | 0 > \epsilon \sqrt{\epsilon}$ .

With (9) and (10) we get

$$\Lambda = \frac{\int_0^{E_c} s \rho_0(s) e^{-\frac{s}{T}} ds}{\int_0^{E_c} \rho_0(s) e^{-\frac{s}{T}} ds}, \quad (11)$$

$$\frac{F^2}{4} (\Lambda_1 + \frac{\Lambda^2}{4}) = \int_0^{E_c} \rho_1(\epsilon) (\epsilon - \Lambda) e^{-\frac{\epsilon - \Lambda}{T}} d\epsilon, \quad (12)$$

$$\frac{F_1 F}{2} = \int_0^{E_c} \rho_1(\epsilon) (1 + \frac{\epsilon - \Lambda}{T}) e^{-\frac{\epsilon - \Lambda}{T}} d\epsilon. \quad (13)$$

Requiring the absolute value of the gluon condensate be less than 30% of the leading perturbative term with continuum subtracted we get the lower limit of the Borel parameter  $T$  and the continuum threshold  $E_c$ . Typically in our analysis the gluon condensate contribution in the whole sum rule is less than 25% for  $1^{-+}$  and 10% for the other three states starting from  $T > 0.9 \text{ GeV}$ . We have kept four active flavors and let  $\Lambda_{\text{QCD}} = 220 \text{ MeV}$ . Varying  $\Lambda_{\text{QCD}}$  from 220 MeV to 300 MeV the final result changes within 5%. We want to emphasize that all the sum rules for  $\Lambda, \Lambda_1, F, F_1$  are stable in the large interval  $0.9 < T < 3.0 \text{ GeV}$ .

As a consistency check we have also fitted the left and right hand side of Eq. (9) directly with the most suitable parameters  $\Lambda, F, E_c$  directly in the working region of the Borel parameter. The results agree very well with those derived from the derivative method.

We collect our final results in TABLE II. For  $\Lambda$  there is an error about  $(\pm 0.2) \text{ GeV}$ . For the exotic state  $H_1$  we present the variations of  $\Lambda$  with  $T$  and  $E_c$  in FIG. 1. We also show the left and right hand side of (9) with the central values of  $\Lambda, F, E_c$  in TABLE II for  $H_1$ . Both sides agree within one percent in the region  $0.5 < T < 3.0 \text{ GeV}$  as can be seen from FIG. 2.

In the limit  $m_Q \rightarrow \infty$  the  $0^{--}$  and  $0^{++}$  hybrid quarkonium is exactly degenerate while the  $\mathcal{O}(1/m_Q)$  correction makes  $0^{--}$  mass lower than that for  $0^{++}$ . The  $1^{-+}$  and  $1^{+-}$  hybrid is nearly degenerate and the  $\mathcal{O}(1/m_Q)$  correction does not split their masses. The  $1^{-+}$  hybrid lies 0.7 GeV lower than the  $0^{--}$  hybrid. The mass of the lowest exotic hybrid quarkonium  $b\bar{b}g$  is  $(10.75 \pm 0.20)$  GeV if we use  $m_b = 4.8\text{GeV}$ .

In summary we have calculated the binding energy of the hybrid quarkonium up to the order of  $\mathcal{O}(1/m_Q)$ . We have found that the decay constants of the hybrid mesons scale like  $m_Q^{\frac{5}{4}}$  as  $m_Q \rightarrow \infty$ . Moreover, the scale of the binding energy is solely set by the gluon condensate. In the sum rules for the light hybrids the four quark condensate and three gluon condensate are not precisely known, which renders the extraction of the light hybrid mass rather difficult [7, 8]. In our calculation of hybrid quarkonium masses we have not considered the error due to the bottom quark mass. Another possible source of error is the lack of  $E_c$  stability.

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## References

- [1] D.R.Thompson et al., Phys. Rev. Lett. 79 (1997) 1630.
- [2] A.Abele et al., Phys. Lett. B 423 (1998) 175.
- [3] D.Alde et al., Phys. Lett. B 205 (1988) 397.
- [4] H.Aoyagi et al., Phys. Lett. B 314 (1993) 246.
- [5] G.M.Beladidze et al., Phys. Lett. B 313 (1993) 276.
- [6] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 147 (1979) 385.
- [7] I.I.Balitsky, D.I.Dyakanov and A.V.Yung, Z. Phys. C 33 (1986) 265.
- [8] J.I.Latorre, P.Pascual and S.Narison, Z. Phys. C 34 (1987) 347.
- [9] J.Govaerts et al., Nucl. Phys. B 248 (1984) 1.
- [10] J.Govaerts et al., Nucl. Phys. B 258 (1985) 215; *ibid.* B 262 (1985) 575; B 284 (1987) 674.
- [11] Shi-Lin Zhu, hep-ph/9812405, Phys. Rev. D (in press).
- [12] L.J.Reinders, H.R.Rubinstein and S.Yazaki, Phys. Rep. 127 (1985) 1.

## Figure Captions

FIG. 1 The variations of  $\Lambda$  with  $T$  and  $E_c$  for  $H_1$ .  $T$  is in unit of GeV.

FIG. 2 The variation of the right and left hand side of Eq. (9) with  $T$  is plotted as solid and dotted curves respectively for  $H_1$  with the values in TABLE II.

TABLE I. The coefficients  $a_i$  etc in  $\rho_0(\epsilon)$  and  $\rho_1(\epsilon)$ .

	$1^{-+}$	$0^{++}$	$1^{+-}$	$0^{--}$
a	$\frac{256}{945}$	$\frac{384}{945}$	$\frac{128}{405}$	$\frac{256}{945}$
b	$\frac{1}{24}$	$-\frac{1}{16}$	$\frac{1}{144}$	$-\frac{1}{24}$
c	$-\frac{128}{945}$	$-\frac{3648}{10395}$	$-\frac{1216}{10395}$	$-\frac{2176}{10395}$
d	$\frac{1}{144}$	$-\frac{1}{96}$	$\frac{7}{288}$	$-\frac{1}{48}$

TABLE II. The values of  $\Lambda, \Lambda_1, F, F_1, E_c$  for the hybrid quarkonium.  $\Lambda, E_c$  is unit of GeV,  $\Lambda_1$  is unit of  $\text{GeV}^2$ ,  $F$  is in unit of  $\text{GeV}^{\frac{11}{4}}$  and  $F_1$  is in unit of  $\text{GeV}^{\frac{15}{4}}$ .

	$1^{-+}$	$0^{++}$	$1^{+-}$	$0^{--}$
$\Lambda$	1.22	1.97	1.21	1.97
$\Lambda_1$	-0.44	-0.44	-0.42	-0.92
$F$	0.244	0.89	0.208	0.515
$F_1$	-0.064	-0.105	-0.036	-0.42
$E_c$	1.6	2.3	1.5	2.3

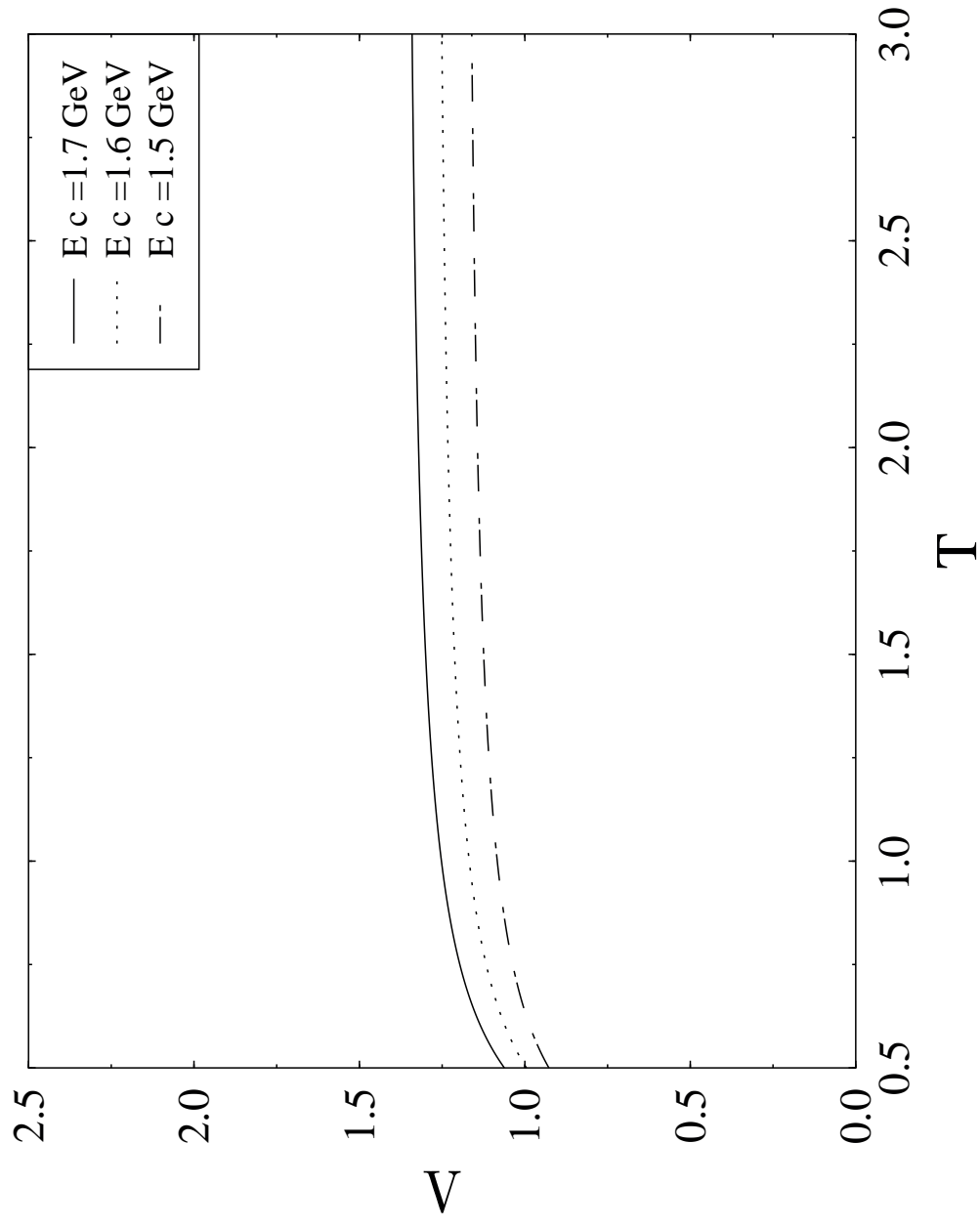


FIG. 1

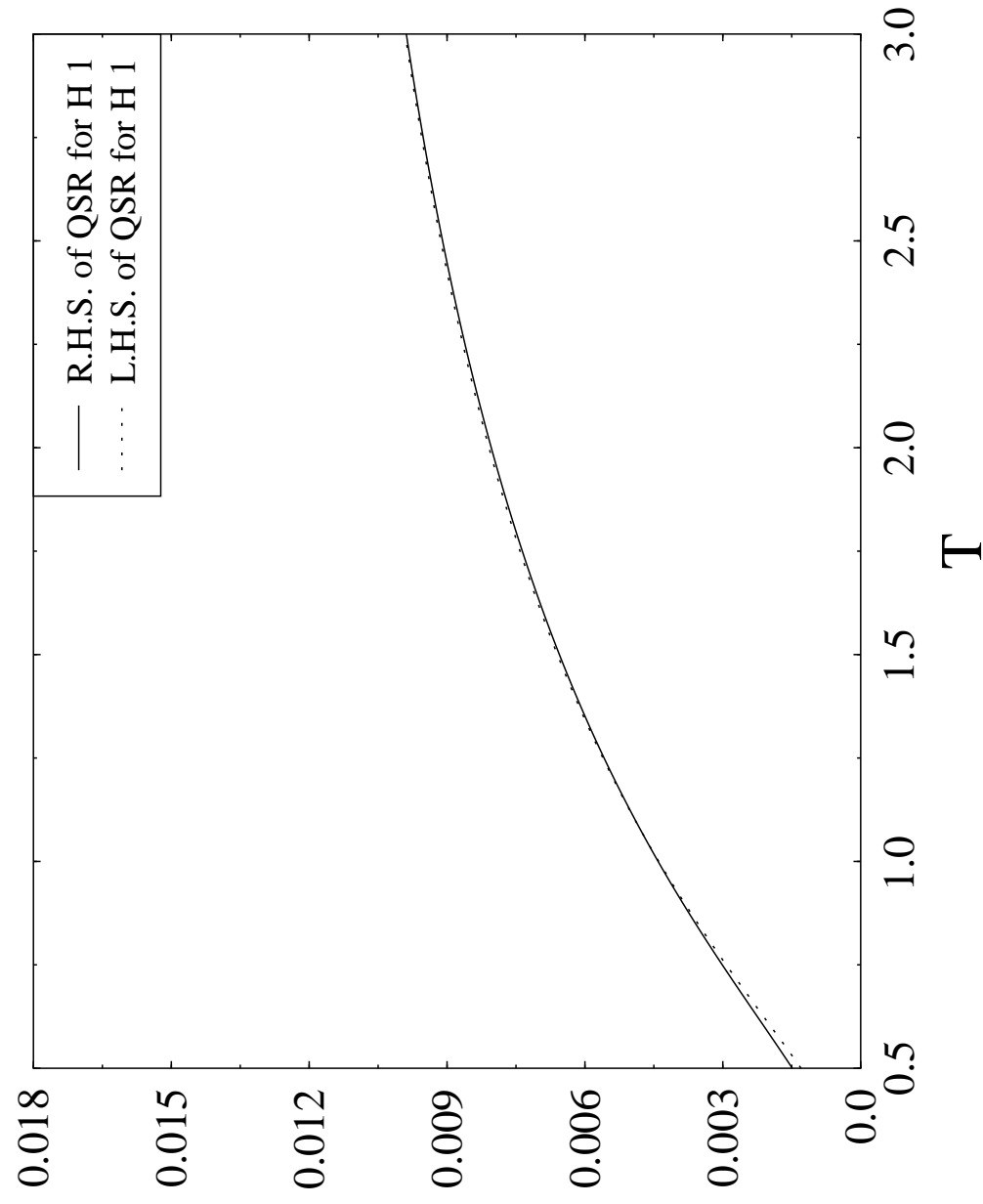


FIG. 2